$$[dN/dX]*VX+[dN/dt]+RC=0 \qquad (12)$$

It is always desirable to put a differential equation into an integral form (no derivatives). It is also necessary to deal with the perfectly reflecting boundaries. Consider the following techniques:

Let there be a differential cell in phase space at a given X, VX and TO (VY was ignored-no dependency in Y or Z exists). Let the interaction cross-section be equal to zero. Then at incremental time DT later, the particles in our differential cell will be in the differential cell at the point:

$$XA=X+VX*DT \qquad (13)$$

$$VXA=VX \qquad (14)$$

Furthermore, since the Jacobian of (13) and (14) is one, the density at the new point (XA, VXA,T+DT) will be the same as the density at the old point (X,VX,T). Hence, using the moving coordinate system, the Lliouville equation becomes:

$$[dN/dt]=-RC \qquad (15)$$

Equation (15) can be readily solved with simple integration.

The problem of the reflective boundary condition can be similarly solved. Upon contact with the boundary, the velocity of the differential cell is reversed and the particles proceed back into the region of interest. Contact with the left-hand boundary results in the following transformation:

$$XA=-2-X-VX*DT \qquad (16)$$

$$VXA=-VX \qquad (17)$$

where $-1 > X+VX*DT$ and contact with the right hand boundary results in the following transformation:

$$XA=2-X-VX*DT \qquad (18)$$

$$VXA=-VX \qquad (19)$$

where $1 < X+VX*DT$. The Jacobian of (16), (17) and the Jacobian of (18), (19) are both unity.

LOGARITHMIC INTEGRATION

Recall from (6) that N=NX*NVX*NVY. Then,

$$[dN/dT]/N=[dNX/dT]/NX+[dNVX/dT]/NVX+ +[dNVY/dT]/NVY \qquad (20)$$

Integration will be performed in moving coordinates where changes in the density function are due entirely to the interaction. Hence, [dNX/dT]=0 and (20) becomes:

$$[dN/dT]/N=RVX/NVX+RVY/NVY \qquad (21)$$

Recognizing that [dN/dT]/N=[d(log(N))/dT], the equation may be readily solved by transforming RVX/NVX and RVY/NVY into moving coordinates by (13), (14), (16), (17), (18), and (19); integrating according to (20); applying the antilog; then transforming the result back to stationary coordinates. Again to simplify the iteration process, operators were used to perform the transformation process.

PRESENTATION OF RESULTS

GENERAL PERFORMANCE

A sequence of computer runs were made to solve the Lliouville equation for a time period of 2.5 milliseconds. It was found that convergence could be achieved for a time slice of 250 microseconds. As a result, ten time slices were run. Convergence was fast; differences of less than 0.1% between successive iterations were achieved in no more than seven iterations. 1.5 hours was required per iteration.

ACOUSTICAL DAMPING

The fluidic kinetic and potential energy functions were computed for the gas as a whole. Kinetic energy was computed from the mean velocity and potential energy was computed from the effective change in volume represented by the changes in density. These functions were integrated over X to obtain the total fluidic kinetic and potential energy in the container. Finally, the total acoustical energy is presented in Fig. 1 and its log is presented in Fig. 2. Notice that the acoustical energy decays exponentially over about a 16 db range. However, notice also that the curve bottoms out; the acoustical energy fails to decay below 0.2 joules.

POSSIBLE EXPLANATIONS

The results of this experiment posed many questions for further study. An always present questions is the accuracy of the approximation (automatic error correcting systems are currently being explored). The following paragraphs present these questions.

Errors in the computation of the RVX and RVY functions were observed. The computation of a derivative is always poor. Under equilibrium conditions (NVX equals NVY), RVX and RVY will be zero. In tests with two Gaussian distributions, a null in the rate was observed where NVX and NVY were equal; however, this null was not equal to zero. Fig. 3 presents RVX at the null. The rate function at equilibrium was such that the temperature of the gas would decrease with time. Fig. 4 presents the total thermal energy of the gas as a function of time. We observe that the total thermal energy does decrease with time. I am presently developing a better algorithm for the computation of this rate.

Another possibility is that the results found are essentially correct. Preliminary computations of the damping factor from classical linearized acoustics reveals a much lower damping factor than observed in Fig. 2. In this experiment, the gas was severly disturbed (10% of the density) and hence linearized acoustics do not hold. It is possible that the "flat region" is the acoustical energy curve of a linearized oscillator. As soon as a rate function with a better null is developed, I will rerun the experiment for a longer period of time.

Another possibility is that the separation of N(X,VX,VY,T) into NX(X,T), NVX(X,VX,T), and NVY(X,VY,T) is not valid due to the presence of cross-terms (VX*VY). A small (1%) cross-term component was found. I believe that these cross-terms are quite significant and plan to investigate them in the near future.

Another possibility is that the use of a potential energy function based on the change of density of the gas is not valid. Preliminary investigations have suggested that such a potential energy function does not exist in this oscillator. The oscillator, instead, appears to be defined by exchange between a mean-velocity energy term and a thermal energy term. In the next set of Lliouville experiments, I will extract the thermal energy term and compare with the mean-velocity energy term.

CONCLUSIONS

The personal computer should be considered as a serious tool of investigation. In the author's first steps as an investigator of statistical mechanics, his Heath H89 computer provided him with many valuable insights and experiences. In the beginning of a project, almost any personal computer equipped with the proper system can provide a low cost means of preparing the project for presentation to a sponsoring agency. In many cases, it may eliminate the need to solicit sponsorship prior to starting the